A Subcell Remapping Method on Staggered Polygonal Grids for Arbitrary-Lagrangian-Eulerian Methods

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In this work we have constructed a full 2D remapping method to be used on a staggered polygonal mesh. This technique has been implemented into an Arbitrary-Lagrangian-Eulerian (ALE) code. It combines and generalizes previous work on the Lagrangian and rezoning phases including this new remapping algorithm [1]. In the Lagrangian phase of the ALE method we use compatible methods to derive the discretizations [2, 3]. We assume a staggered grid where velocity is defined at the nodes, and where density and internal energy are defined at cell centers. In addition to nodal and cell-centered quantities, our discretization employs subcell masses that serve to introduce special forces that prevent artificial grid distortion (hourglass-type motions) [4]. This kind of numerical scheme adds an additional requirement to the remap phase: that the subcell densities (corresponding to subcell masses) have to be conservatively interpolated in addition to nodal velocities and cell-centered densities and internal energy. In the remap phase, we assume that the rezone algorithm produces a mesh that is "close" to the Lagrangian mesh so that a local remapping algorithm (i.e., where mass and other conserved quantities are only exchanged between neighboring cells) can be used.

Our new remapping algorithm consists of three stages.

- A gathering stage, where we define momentum, internal energy, and kinetic energy in the subcells in a conservative way such that the corresponding total quantities in the cell are the same as at the end of the Lagrangian phase.
- A subcell remapping stage, where we conservatively remap mass, momentum, internal, and kinetic energy from the subcells of the Lagrangian mesh to the subcells of the new rezoned mesh.
- A scattering stage, where we conservatively recover the primary variables: subcell density, nodal velocity, and cell-centered specific internal energy on the new rezoned mesh.

We have proved that our new remapping algorithm is conservative (in mass, momentum, and total energy), reversible (if the old and new meshes are identical then the primitive variables are kept unchanged), at least positive (density and specific internal energy are kept positive thanks to a repair method, at most, preserving a maximum principle), and satisfies the DeBar consistency condition (if a body has a uniform velocity and spatially varying density, then the remapping process should exactly reproduce a uniform velocity). We have also demonstrated computationally that our new remapping method is robust and accurate for a series of test problems in one and two dimensions. Figure 1 presents the results of the Sedov blastwave in 2D Cartesian coordinates for a polygonal mesh in ALE regime: a cylindrical shock wave is initiated at the origin and at $t = 1.0$ its exact location is $r = 1$. In this run the rezone strategy improves the mesh quality and the remapping technique preserves the accuracy of the Lagrangian scheme without the its pathological behaviors.

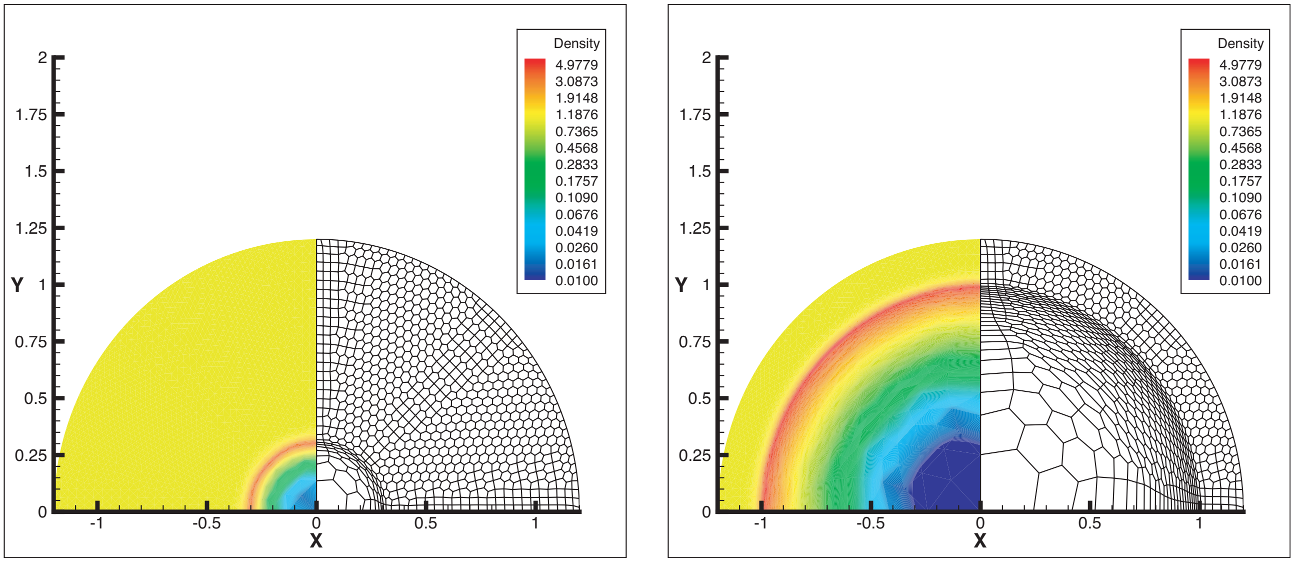

Figure 1—
Sedov blastwave on a polygonal mesh (1325 nodes and 775 cells)—ALE-10—regime-mesh and density contours (exponential scale) at t = 0.1, and t = 1.0.

[1] R. Loubere, M. Shashkov, "A Subcell Remapping Method on Staggered Polygonal Grids for Arbitrary-Lagrangian-Eulerian Methods," Los Alamos National Laboratory report LA-UR-04-6692 (September 2004), submitted to *J. Comp. Phys.*
[2] J. Campbell and M. Shashkov, "A Compatible Lagrangian Hydrodynamics Algorithm for Unstructured Grids," *Selcuk J. Appl. Math.*, **4** (2003), pp. 53–70; report version can be found at (http://cnls.lanl.gov~shashkov).
[3] J. Campbell, M. Shashkov, "A Tensor Artificial Viscosity using a Mimetic Finite Difference Algorithm," *J. Comp. Phys.*, **172** (2001), pp. 739–765.
[4] E. J. Caramana and M. J. Shashkov, "Elimination of Artificial Grid Distortion and Hourglass-Type Motions by means of Lagrangian Subzonal Masses and Pressures," *J. Comp. Phys.*, **142** (1998), pp. 521–561.

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